

國立中興大學105學年度碩士班招生考試試題

科目：工程數學

系所：土木工程學系甲乙組

本科目可以使用計算機

本科目試題共 / 頁

1. Solve the following ODEs.

(15%)

(a) $D^3(D^2 - 4)(D^2 + 3D + 4)y = 0$, where differential operator $D = \frac{d}{dt}$.

(b) $x^2y'' - 2xy' + 2y = 0$, $y(1)=3$, $y'(1)=2$.

2. (a) Describe the convolution theorem of Laplace transforms.

(15%)

(b) Using (a) and the other method to find inverse Laplace transform of $\frac{1}{s(s^2+1)}$.

3. Let a matrix $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$, to find

(25%)

(a) The sum of a symmetric matrix and a skew-symmetric matrix equals to A .

(b) $|A^{21}|$, and A^{-1} (if it exist).

(c) A similarity transformation to diagonalize A and the diagonal matrix.

4. Using half-range expansion method to expand the equation:

$f(x) = x$, for $0 < x < L$,

(10%)

(a) in a cosine series, (b) in a sine series

5. Solve the following partial differential equation:

(20%)

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} - G$$

with two boundary conditions: $u(0, t) = 0$ and $u(L, t) = 0$;

and two initial conditions: $u(x, 0) = f(x)$ and $u_t(x, 0) = g(x)$.

Also interpret the physical meanings of eigenvalues and eigenfunctions of this equation.

6.(a) Find the Maclaurin series of $f(z) = 1/(1 + z^2)$,

(5%)

(b) Evaluate $\int_{-\infty}^{\infty} \frac{\sin mx}{x(x^2+k^2)} dx$ ($m \geq 0, k > 0$)

(10%)