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Modeling Water Erosion Due to Overland Flow Using Physical Principles 1. Sheet Flow

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A new model for erosion of plane soil surfaces by water is developed using physical principles. Raindrop impact and overland flow remove soil from the original cohesive soil. Once eroded soil enters overland flow, either as aggregates or primary particles, a significant proportion of it returns to the soil bed, forming a cohesionless deposited layer from which it can be removed again by the same erosion processes. The action of the eroding agents will be divided between eroding the unshielded original cohesive soil and reintroducing sediment from the deposited layer. The theory recognizes that the nature of the surface is modified by the erosion and deposition processes affecting it. Solutions of the governing differential equations describing sediment concentration are developed for two distinct equilibrium cases. The first case, when the deposited layer completely shields the original soil, appears to correspond with what has been previously called a "transport-limited" situation. The second case occurs when such shielding is incomplete, and sediment concentration is affected by the cohesive strength of the soil. The resulting equations for sediment concentration at equilibrium are compared with existing equations. Firstly, the equation for the case where the soil is lacking cohesion is shown to be similar to the semiempirical equation of Yang (1973). Secondly, when the soil is cohesive the slope length relationships are shown to be in good agreement with the universal soil loss equation over a wide range of slope steepness.

INTRODUCTION

It has been widely supposed that the soil erosion processes occurring during overland flow are very similar to those occurring during streambed erosion. This has led to the use of sediment transport equations derived for deep flow conditions to describe the movement of sediment in the relatively shallow flows characteristic of soil erosion on a field scale. However, there are differences between these two scales in both the sedimentary materials and the processes at work. Firstly, the sediment which is being acted upon at the field scale is generally cohesive, having both interaggregate and interparticle strength in situ. Secondly, soils are commonly composed of a wide range of aggregate and particle sizes. Finally, the shallow surface flows which occur at field scales are influenced by the impact of raindrops on both the shallow water layer and the exposed soil surface. In contrast, streambed erosion is characterized by well-sorted cohesionless sediment which is carried in relatively deep flows uninfluenced by raindrop impact.

The influence of soil physical properties on the erosion process has been recognized through the concept of soil "erodibility," which is reflected in the ratio of actual sediment flux to the sediment flux expected for cohesionless single-sized sediment. It has long been recognized that there are three distinct phases in the movement of a sedimentary unit from one point to another: detachment, transport and deposition. Clearly, only initial removal is dependent upon the cohesive resistance of the original soil. *Foster and Meyer* [1975] introduced two terms, "detachment limiting" and "transport limiting," to describe sediment flux when either the resistance to the original soil to release sediment, or the ability of overland flow to move sediment are respectively considered limiting. Streambed sediment transport equations have been employed to describe the transport-limiting case, and empirically derived erodibilities have been used for the detachment-limiting case.

In this paper, two limiting cases are considered. First, the case when soil strength limits the sediment concentration is described by assuming that a certain amount of energy is required to remove a unit mass of cohesive soil (called the "specific energy of entrainment"). The second case considered is the transport-limiting instance, where a cohesionless layer of deposited sediment completely shields the original soil surface. In both cases, some fraction (not all) of the stream power of overland flow is assumed to be consumed in removing sediment. The contribution of rainfall impact to sediment concentration has been considered separately [*Hairsine and Rose*, 1991] and while compatible with this approach, it is not considered in this paper.

In this paper, overland flow is considered to be evenly distributed across the slope and behaves according to the approximate kinematic wave model proposed by *Rose et al.* [1983*a*]. In paper 2 [*Hairsine and Rose*, this issue], the influence of flow concentrations, such as rills, is considered and the resulting theory tested using experimental data obtained using simulated rainfall.

FORMATION OF A DEPOSITED LAYER

Whenever sediment with a positive immersed weight exists within a flow, all but the finer suspended sediment has a velocity component tending to move it downward under the action of gravity. A continuous deposition process results which may be described [*Croley*, 1982] by the expression

$$d_i = \alpha_i v_i c_i, \tag{1}$$

where i refers to a general settling velocity or size class, d_i is the mass rate of deposition per unit area of that class,

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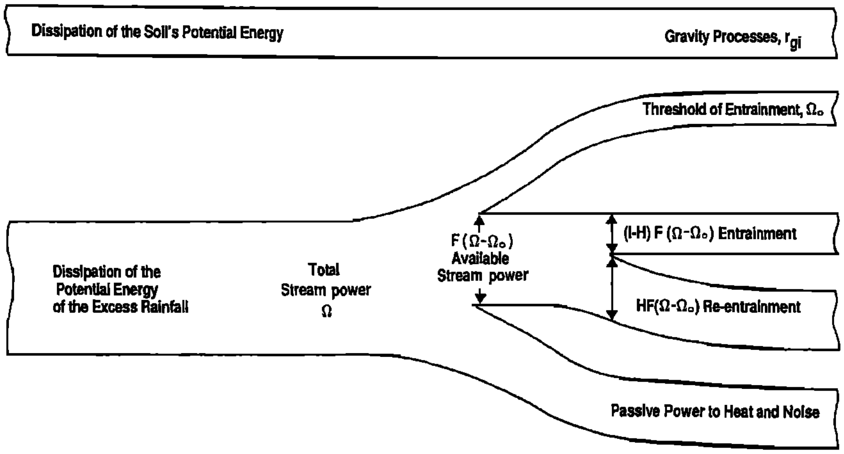

Fig. 1. Flow diagram of the use of the potential energy of the excess rainfall and the soil mass itself, with sources on the left and sinks on the right.

$(\alpha_i c_i)$ is the sediment concentration adjacent to the bed, c_i is the mean sediment concentration (mass of sediment per unit volume of solution), and v_i is the settling velocity representative of class i. Note that the term α_i is introduced to permit a nonuniform vertical distribution of sediment in the flow. This term does not need to be evaluated for sediment transport at equilibrium as will be shown below.

Sediment returned to the bed is considered available for removal by the actions of overland flow (or raindrop impact). Thus, a dynamically changing deposited layer is conceived as being continuously added to by deposition and removed by "reentrainment." Both entrainment and reentrainment are due to the mutual shear stress between the surface and the flow, but reentrainment describes removal of sediment from the deposited layer. The relative extent of the deposited layer in protecting the underlying soil at any time is denoted by H, the fractional shielding of the original soil surface by the deposited layer. This fractional surface shielding could include a contribution by contact load developed by erosion, though this will not be dealt with explicitly. Fractional exposure of the original cohesive soil to entrainment is then $(1 - H)$. As the deposited layer is added to by deposition only, it is assumed it will be noncohesive. Thus it will be more readily removed than the original uneroded soil.

PROCESS RATE EQUATIONS

The rate of working of the mutual shear stress between the soil surface and overland flow was called the stream power (Ω) by *Bagnold* [1966]. Stream power is the power per unit bed area available to do work. With the flow uniformly distributed across a plane of slope S, stream power is given by

$$\Omega = \rho g S q \tag{2}$$

where ρ is the water density; g, the acceleration due to gravity; and q, the water flux per unit width. *Rose et al.* [1983*b*] assumed a threshold stream power (Ω_0), below which no soil was entrained (the term used to describe removal of original soil by overland flow) or reentrained (the removal of sediment deposited in the current event by overland flow).

Not all the excess stream power $(\Omega - \Omega_0)$ is used to erode soil; some is dissipated as heat and noise. Suppose fraction F of $(\Omega - \Omega_0)$ is effective in entrainment or reentrainment. We then assume that stream power is applied uniformly to the total wetted perimeter of flow. However, where the deposited layer shields the original soil, stream power applies only to the deposited layer. Thus, the rate of reentrainment is driven by the effective excess stream power, $HF(\Omega - \Omega_0)$, and the rate of entrainment by the remaining effective excess stream power, $(1 - H)F(\Omega - \Omega_0)$. The sources and sinks of stream power in overland flow are shown in Figure 1. Note that the dissipation of the potential energy of the soil itself has the potential to contribute to the rate at which soil is introduced to the flow. The contribution of head cut collapses and slumping of rill walls is described by the gravity process rate, r_{gi} (kilograms per square meter per second). In the extreme case of land slips and mudflows, this process would dominate. However, on more moderate slopes, its role is limited to contributing sediment to the flow which then may be acted on by deposition and reentrainment.

The Process of Sediment Entrainment

Entrainment is the term adopted to describe the removal of sediment from the original cohesive soil mass or soil matrix by the action of overland flow. Any strength possessed by this soil matrix will result in there being some resistance offered to such entrainment. Because entrainment is from the soil surface where there is negligible overburden pressure, any soil strength must be dominantly due to cohesion. *Raudkivi and Tan* [1984] showed experimentally that (for cohesive sediment) the effects of cohesion are far greater than that of the submerged weight of detached particles in inhibiting removal of soil. The resistance offered by the soil matrix to removal by fluid stresses exerted on it by overland flow is defined by the energy per unit mass of soil required to entrain it, J, which is called the specific energy of entrainment.

Cohesive strength is a property of the soil matrix as a whole, not of individual particles. Thus, when cohesive strength is overcome in soil removal by entrainment, this process is assumed not to be size selective. If the original soil aggregates break down on wetting into a particular distribution of size or settling velocity classes, then the process of entrainment is assumed not to change this distribution. This distribution, which was called a settling velocity distribution by *Lovell and Rose* [1988], is divided into I classes each of equal mass. The rate of entrainment (r_i, kilograms per square meter per second) of any general size class i, will be equal for all I classes from the assumed nonspecificity with respect to size. The total rate of entrainment is then Ir_i. Thus, the total rate of energy expenditure on entrainment is Ir_iJ, which must be equal to the effective excess stream power available to sustain this process, given above as $(1 - H)F(\Omega - \Omega_0)$. It follows that

$$r_i = (1 - H)\frac{F}{IJ}(\Omega - \Omega_0) \qquad \Omega > \Omega_0 \tag{3}$$

$$r_i = 0 \qquad \Omega \leq \Omega_0. \tag{4}$$

No field test exists for direct measurement of the specific energy of entrainment, so that the value of J must be derived from erosion experiments. However, both the fall cone technique of *Bradford and Grossman* [1982] and the soil-shearing device of *Sargunam et al.* [1973] may provide measurements related to J.

The Process of Sediment Reentrainment

Reentrainment results from the action of overland flow on sediment deposited during the current erosion event on the soil surface. The cohesive strength of this recently deposited sediment is assumed to be insignificant, so that the force resisting removal by the flow depends solely on the immersed weight of sediment. The power expended in lifting the sediment to some height in the flow may therefore be assessed through the rate of change in potential energy of this sediment. Denote the rate of reentrainment (in mass per unit area of bed per second) by r_{ri} for sediment of size class i. Assuming the sediment density σ is the same for all size classes, its immersed weight is proportional to $(\sigma - \rho)/\sigma$. An alternative description of the term α_i introduced with (1) is that D/α_i is the height through which sediment is lifted in reentrainment. Hence, the power required per unit area of bed to reentrain sediment of size class i to height D/α_i is $r_{ri}g(D/\alpha_i)(\sigma - \rho)/\sigma$.

This power may be equated to the power available for reentrainment, $HF(\Omega - \Omega_0)$. As with entrainment, reentrainment is assumed to be nonselective with respect to sediment size of the source material, in this case the deposited layer. Therefore, the fraction of the total rate of reentrainment for each size class is proportional to the mass fraction of that class in the deposited layer. This is given by M_{di}/M_{dt}, where M_{di} is the mass of sediment of class i in the deposited layer per unit area of bed, and M_{dt} is the total mass of the deposited layer per unit area. Equating the expressions given above for the power requirement and power available per unit area for reentrainment, the expression for the rate of reentrainment becomes

$$r_{ri} = \frac{\alpha_i HF}{g}\frac{\sigma}{(\sigma - \rho)}\left(\frac{\Omega - \Omega_0}{D}\right)\frac{M_{di}}{M_{dt}} \qquad \Omega > \Omega_0 \tag{5}$$

$$r_{ri} = 0 \qquad \Omega \leq \Omega_0. \tag{6}$$

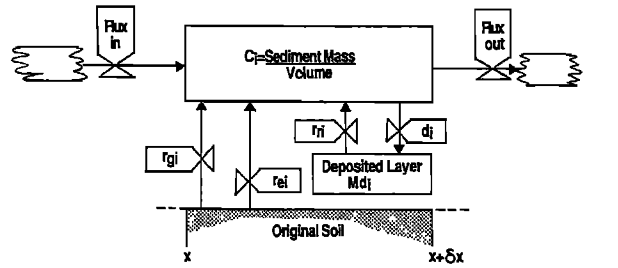

Fig. 2. Flow diagram (after the style of *Forrester* [1970]) describing the interaction of erosion processes between the sediment flux, the original soil and the deposited layer. The process rates are entrainment, r_{ei}, reentrainment, r_{ri}, gravity processes, r_{gi}, and deposition, d_i.

The Processes of Rainfall Detachment and Rainfall Redetachment

Hairsine and Rose [1991] distinguished between the processes of rainfall detachment and rainfall redetachment. Both processes are driven by the impact of raindrops on the water-covered soil surface, detaching original cohesive soil and redetaching sediment from the deposited layer. The rate of sediment addition to the water layer by rainfall detachment and rainfall redetachment is reduced as flow depth increases [*Proffitt et al.*, 1991] for water depths greater than about three drop diameters. These processes may be small contributors to sediment concentration compared with entrainment and reentrainment. For analytical simplicity, the solutions developed below are for the case when flow depths are such that the contributions of these processes can be ignored. However, using numerical methods, solution of the differential equation describing conservation of mass of sediment with both rainfall- and runoff-driven processes can be obtained [*Hairsine*, 1988].

SEDIMENT CONCENTRATION AT EQUILIBRIUM

Consider sediment transport on a plane land element with volumetric water flux per unit width q and depth of flow D. If rainfall detachment and redetachment rates are negligibly small, then from Figure 2, mass conservation of sediment in the general settling velocity class i requires that

$$\frac{\partial(qc_i)}{\partial x} + \frac{\partial(c_iD)}{\partial t} = r_i + r_{ri} + r_{gi} - d_i, \tag{7}$$

where x is distance downslope and t is time. The flowchart in Figure 2 represents the four processes affecting sediment concentration and thus sediment flux. Equation (7) needs to be accompanied by a description of the hydrology of overland flow. Using the approximate analytic solution of *Rose et al.* [1983*a*] gives

$$q = Qx, \tag{8}$$

where Q is the runoff rate per unit area, and using the kinematic flow approximation gives

$$q = KD^m. \tag{9}$$

Manning's and Chezy's equations are specific forms of (9) for turbulent flow. In Manning's equation, $m = 5/3$ and $K = S^{1/2}/n$, where n is Manning's roughness coefficient. For Chezy's equation, $m = 3/2$ and $K = CS^{1/2}$, where C is experimentally determined. For overland flow in the presence of rainfall, it is expected that the character of the flow will be turbulent. For turbulent flows, Manning's equation will apply, with m approximately 5/3 [*Overton and Meadows*, 1976]. However, theory developed in this paper will use (9), permitting K and m to be chosen as desired.

Mass conservation of sediment in the deposited layer alone (Figure 2) also requires that

$$\frac{\partial M_{di}}{\partial t} = d_i - r_{ri}. \tag{10}$$

General solutions of (7)–(10) require description of the gravity process, r_{gi}, and use of numerical techniques. The solution presented below is restricted to equilibrium situations when water depth, the mass of the deposited layer, and the concentration of each of the settling velocity classes do not change with time. That is, dynamic equilibrium is assumed where all variables are independent of time. *Proffitt and Rose* [1991] showed that for steady flow conditions a dynamic equilibrium was approached for both rain- and flow-driven erosion, and that equilibrium was associated with the settling velocity distribution of the eroded sediment approaching that of the original soil. This observation is consistent with the equilibrium solutions developed below.

Equilibrium conditions yield simple analytical solutions which provide upper and lower limits to the sediment concentration. Furthermore, in erosion events where the above variables are not changing rapidly, it is possible that the use of equilibrium solutions in which input rates such as rainfall rate are allowed to adopt a sequential series of time-averaged values may well provide a solution of sufficient accuracy for events where change is not too rapid.

After equilibrium has been achieved, the rate of reentrainment from, and deposition to the deposited layer, will be equal, so the mass of the deposited layer is constant (i.e., $\partial M_{di}/\partial t = 0$, equation (10)). The shielding, H, of the original soil matrix by this deposited layer will be constant at equilibrium, and may be either complete ($H = 1$) or partial ($H < 1$). These two cases will be examined below.

The Entrainment-Limiting Case (H < 1)

With $H < 1$ and the mass per unit area of the deposited layer steady, a constant fraction $(1 - H)$ of the original soil is exposed to the erosive action of overland flow. From (10), with $\partial M_{di}/\partial t = 0$, the processes of reentrainment and deposition are in equilibrium. If rainfall detachment and gravity processes are negligible, it then follows from Figure 2 that only entrainment of the original soil causes the sediment flux to change with downslope distance. As the rate at which both rainfall detachment and entrainment processes act is controlled by the strength of the original soil, this case where $H < 1$ may be termed "source limiting" or, in the situation assumed here where entrainment is the only process, "entrainment limiting." This concept is very similar to that of "detachment limiting" proposed by *Foster and Meyer* [1975] for the situation when the strength of the original soil controls the rate of change of sediment flux. A difference is that the "source limit," and in particular the "entrainment limit" derived here, is explicitly described as for the situation when the mass of the deposited layer is constant and the shielding by that layer is less than unity. Therefore, at the source limit, the value of the shielding fraction, H has a direct impact on the rate at which the processes of entrainment and rainfall detachment act. The concept of a deposited layer is not used by *Foster and Meyer* [1975].

Substituting from (1) for d_i and (5) for r_{ri} into (10), taking $\partial M_{di}/\partial t = 0$, summing over all I size classes, and recognizing that $c = \sum_{i=1}^{I} c_i$ and $\sum_{i=1}^{I} M_{di}/M_{dt} = 1$, leads to the following expression for the shielding (H) of the deposited layer:

$$H = \frac{cgD[(\sigma - \rho)/\sigma] \sum_{i=1}^{I} v_i/I}{F(\Omega - \Omega_0)}. \tag{11}$$

Equation (11) followed from the assumption of equality between the rate of deposition to form the deposited layer and reentrainment from it. This same assumed equality of rates at dynamic equilibrium allows (7) to be simplified to

$$q\frac{dc_i}{dx} + c_i\frac{dq}{dx} = r_e. \tag{12}$$

Note that the rate of gravity processes is assumed zero in this entrainment-limiting case, which describes the lower limit for sediment concentration.

Using the boundary condition that the sediment concentration, c, is zero at the threshold of entrainment (when $\Omega = \Omega_0$), then the solution to (12) with (3) for r_i and (11) for H is

$$\begin{aligned} c = {} & \frac{A_1 u^{m-1}}{Y}\left[1 - \frac{(2m-1)}{mYu} + \frac{(2m-1)(2m-2)}{(mYu)^2} - \cdots\right] \\ & - \frac{A_0}{uY}\left[1 - \frac{(m-1)}{mYu} + \frac{(m-1)(m-2)}{(mYu)^2} - \cdots\right] \\ & + A_0 m u^{m-1} e^{mY(u_0 - u)} \\ & \cdot \left[\frac{1}{Yu_0} + \frac{(m-1)(m-2) - (2m-1)(2m-2)}{(mYu_0)^2} \right. \\ & - \frac{(m-1)(m-2)(m-3) - (2m-1)(2m-2)(2m-3)}{(mYu_0)^3} \\ & \left. + \cdots \right], \end{aligned} \tag{13}$$

where

$$u = q^{1/m} \qquad u_0 = q_0^{1/m} \qquad q_0 = \frac{\Omega_0}{\rho g s}$$

$$Y = \frac{\sum_{i=1}^{I} v_i/Ig(\sigma - \rho)}{J\sigma Q K^{1/m}}$$

$$A_1 = \frac{F\rho g S}{QJ} \qquad A_0 = \frac{\Omega_0 F}{QJ}.$$

In (13), the power series are infinite and typically rapidly converging when m is a noninteger, and finite when m is an integer.

The Transport-Limiting Case (H = 1)

We now consider the other equilibrium-limiting situation when the mass of the deposited layer is still constant, but the shielding is complete, so that $H = 1$. Complete coverage of the bed by the deposited layer may be produced in a rill, for example by action of the gravity process rate, as in the case of rill bank collapse. As before, it is assumed that water depth is sufficient for rainfall impact to be a negligibly small contributor to sediment concentration. Thus, sediment concentration is determined by the rates of reentrainment, gravity processes and deposition. Since the original soil matrix is completely shielded, the strength of that matrix cannot affect the sediment concentration achieved. This would be expected to be a maximum because the deposited layer is assumed to offer no resistance to its reentrainment, work being done only in lifting this sediment against its immersed weight. It will be shown below that the equilibrium sediment concentration achieved in this case involves a summation based on the settling velocity characteristics of the eroded sediment.

This case appears to correspond approximately to the "transport-limiting" case of *Foster and Meyer* [1975]. These authors used this term to describe a situation where they considered the sediment concentration to be limited by the ability of the flow to carry a sediment load. *Foster and Meyer* [1975] make no specific reference to sediment deposition, nor to the settling velocity characteristic of eroded sediment in the bed load transport equation adapted by them to describe the sediment flux in this transport-limiting situation. Although the conceptual descriptions and analytic expressions of this situation are not identical to the "transport limit" of *Foster and Meyer* [1975], that term is still used here to describe the equilibrium case of $H = 1$, because of the apparent similarities.

Substituting $\partial M_{di}/\partial t = 0$ and $H = 1$ into (10) with r_{ri} given by (5) and d_i by (1) with $c_i = c/I$ and then summing over all I size ranges yields directly an expression for c:

$$c = \frac{F[\sigma/(\sigma - \rho)](\Omega - \Omega_0)}{gD \sum_{i=1}^{I} v_i/I}. \tag{14}$$

Since $H = 1$, $r_i = 0$ for this case, and also since $r_{ri} = d_i$ is assumed, the right-hand side of (7) is equal to the rate of gravity processes. Neglecting the time derivative on the left-hand side of (7) because of the assumed dynamic equilibrium, then $d(qc_i)/dx$ and so $d(qc)/dx$ would be ascribed to gravity processes in this case. It follows from (14), (2), (8) and (9) that, at the transport limit, neglecting Ω_0,

$$\frac{d(qc)}{dx} = \sum_{i=1}^{I} r_{gi} = \frac{F\rho[\sigma/(\sigma - \rho)]S(2 - 1/m)}{\sum_{i=1}^{I} v_i/I} \cdot K^{1/m}Q^{2-1/m}x^{1-1/m}. \tag{15}$$

Thus, the rate of gravity processes at the transport limit has been derived and shown to increase at a rate proportional to $x^{1-1/m}$. Clearly, gravity processes are discontinuous in nature. Thus, the transport limit must be considered an unstable limit sustained only by gravity process contributions of sediment. In the absence of this rate of gravity processes, the sediment concentration will then approach the source-limiting concentration given by (13).

DISCUSSION

Equations (13) and (14) are respectively expressions for the sediment concentration in a flow across a cohesive soil surface when either the cohesive strength of the original source soil, or the ability of the flow to reentrain sediment in the presence of deposition, is limiting. Both the cohesive nature of the original soil and the settling velocity (or size) distribution of this soil play a role in this description. The division of the processes into those affected and those not affected by the cohesive strength of the soil, and specific representation of the deposited layer, provide the basis of this model of erosion phenomena.

Equation (14), the expression for sediment concentration at the transport limit, uses the soil parameters, the density of the sediment, σ, and the depositability of the original soil, $\Sigma_{i=1}^{I} v_i/I$, which may be measured using the techniques described by *Hairsine and McTainsh* [1986] or *Lovell and Rose* [1988]. Required hydraulic parameters are stream power, Ω, and flow depth, D, which can be obtained by field measurements or a compatible predictive model of overland flow. Model parameters which require field evaluation are the threshold of entrainment, Ω_0 and the fraction of the stream power used by either entrainment or reentrainment, F. Equation (13), the expression for sediment concentration at the source limit, requires the additional parameter the specific energy of entrainment, J, to be determined by fitting with field or rainfall simulator data. *Proffitt* [1988] found the parameters F and J to be consistent across a range of hydraulic conditions for a cohesive soil.

At equilibrium, when the mass of the deposited layer is steady, it follows from (10) that the rates of reentrainment and deposition are equal, and so

$$\alpha_i v_i c_i = \frac{\alpha_i HF\sigma}{g(\sigma - \rho)} \left(\frac{\Omega - \Omega_0}{D}\right) \frac{M_{di}}{M_{dt}} \tag{16}$$

Now, at equilibrium, $c/I = c_i$, where c is the total sediment concentration. Therefore,

$$v_i c/I = \frac{HF\sigma}{g(\sigma - \rho)} \frac{(\Omega - \Omega_0)}{D} \frac{M_{di}}{M_{dt}} \tag{17}$$

Since v_i and M_{di}/M_{dt} are the only class-selective terms in (17) at equilibrium, M_{di}/M_{dt} is proportional to v_i. Also, since by definition $\Sigma_{i=1}^{I} M_{di}/M_{dt} = 1$, by summing across the classes it follows that

$$M_{di}/M_{dt} = v_i \Bigg/ \sum_{i=1}^{I} v_i \tag{18}$$

Typical cohesive aggregated soils have a $\Sigma_{i=1}^{I} v_i/I$ of the order of 0.05 m s^{-1} [*Proffitt et al.*, 1991] and fine sediment has sediment velocities of the order of 10^{-5} m s^{-1}. Thus, from (17), the equilibrium solutions presented in this theory are consistent with the observation that the deposited layer is dominated by coarse sediment, containing negligible amounts of fine sediment. This prediction is consistent with the measurements of *Nouh* [1990] who studied self-armoring of an eroding multisized sand bed.

The expression for the entrainment limit (equation (13)) may be simplified for the special case when the depth discharge exponent $m = 2$ and the threshold of entrainment, Ω_0, is taken as zero. In this case, (13) becomes

$$c = \frac{F\rho S(KQ)^{1/2}\sigma/(\sigma-\rho)x^{1/2}}{\sum_{i=1}^{I} v_i/I} \cdot \left[1 - \frac{3}{2}\phi + \frac{3}{2}\phi^2 - \frac{3}{4}\phi^3\right] \tag{19}$$

where

$$\phi = \frac{J(KQ)^{1/2}\sigma/(\sigma-\rho)}{\sum_{i=1}^{I} v_i/Igx^{1/2}}.$$

It can be seen from (19) that for a soil of very low cohesive strength (i.e., J has a small value), the sediment concentration at the entrainment limit approaches the concentration at the transport limit given by (14). In this case, the change in sediment flux with respect to distance downslope is equal to the rate of entrainment acting over a small area since $(1 - H)$ must also be small. Thus it now has been shown that sediment concentration at the transport limit can be approached in two ways: firstly, for a cohesive soil, by gravity processes acting at a rate given by (15), and secondly, by the specific energy of entrainment of a noncohesive soil approaching zero.

The outcomes of the theory presented above are now compared with the sediment transport equation of *Yang* [1973], which has been used in erosion models such as that of *Moore and Burch* [1986] and the European soil erosion model [*Morgan et al.*, 1989]. For cohesionless beds, it is appropriate to compare Yang's equation with (14), the expression for sediment concentration at the transport limit. Consider flow down a slope of sediment with a single sediment settling velocity (denoted by v). From (14), sediment concentration would then be

$$c = \frac{F}{gv}\left(\frac{\sigma}{\sigma-\rho}\right)\left(\frac{\Omega-\Omega_0}{D}\right). \tag{20}$$

With the flow uniformly distributed across a plane, stream power may be expressed as

$$\Omega = \rho g S D V \tag{21}$$

where V is the mean flow velocity. Substituting for Ω from (21) in (20) gives

$$c = F\rho\left(\frac{\sigma}{\sigma-\rho}\right)\left(\frac{SV}{v} - \frac{\Omega_0}{\rho g D v}\right). \tag{22}$$

Yang [1973] expressed the sediment concentration at transport capacity (or for transport-limiting situations) as

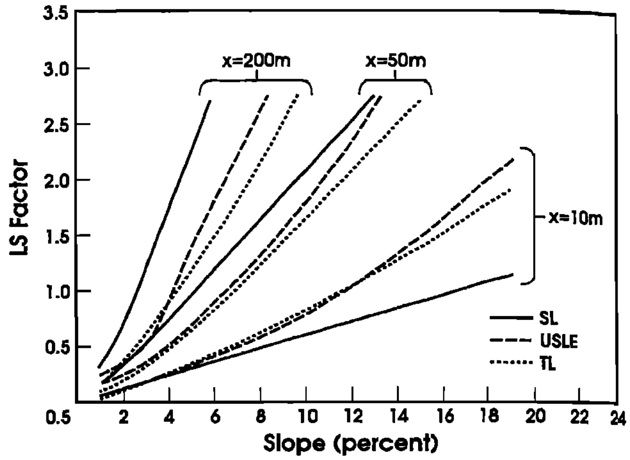

Fig. 3. Comparison of the slope length factor (LS) predictions of the model at the transport limit (TL) and source limit (SL) with those of the universal soil loss equation [*Wischmeier and Smith*, 1978].

$$\log c = X + Y \log\left(\frac{VS - VS_{cr}}{v}\right), \tag{23}$$

where X and Y are constants which are explicitly defined by the sediment size and kinematic viscosity of the water. The subscript cr refers to the critical value of VS at which entrainment begins. Equations (22) and (23) are of the same mathematical form, with the role of unit stream power (VS) and the settling velocity (v) being identical in both equations. The similarities between (22) and (23) suggest there is experimental support from the literature for the form of theory derived above when this is applied to the transport-limiting case for single-sized, noncohesive sediment.

The adequacy of the approach presented in dealing with a range of sediment sizes remains untested against the large body of available streambed data. However, in modeling the erosion and deposition of soils, which have a wide range of both particle sizes and densities, this approach appears to be a conceptual improvement upon the use of a median size and its associated settling velocity commonly adopted in the streambed literature. *Moore and Burch* [1986] suggest that the transport of detached sediment is dominated by a "framework population" representing the saltating particles which are dominantly active. The term $\Sigma_{i=1}^{I} v_i/I$ in this paper is strongly influenced by this saltating fraction of detached sediment.

The influence of slope length and slope steepness upon the rates of erosion is central to soil erosion prediction and associated soil conservation strategies. The very extensive data base summarized in the universal soil loss equation (USLE) [*Wischmeier and Smith*, 1978] contains the largest body of slope length and slope steepness data available. Figure 3 compares the LS factor as predicted by *Wischmeier and Smith* [1978] with that of the theory presented above, for a range of slopes and slope lengths typical of agricultural fields. The parameters required for the theory were taken as $F = 0.245$, $J = 30.5$ J kg^{-1} and $\Sigma_{i=1}^{I} v_i = 0.048$ m s^{-1} from *Hairsine* [1988], who evaluated these parameters for a bare cultivated vertisol from the Darling Downs of Australia. For the three slope lengths considered, the range of slope length factors defined by the source and transport limits shows good agreement with those found by *Wischmeier and Smith* [1978]. For the slope length of 10 m, the transport limit predictions closely follow the USLE relationship. For such slope lengths, the processes of rainfall detachment and rainfall redetachment are likely to make a significant contribution resulting in the sediment concentration's approaching the transport limit.

Paper 2 considers the effect of rill formation on the above theory, and compares theory with experimental data.

CONCLUSIONS

The development of a new model of erosion of cohesive soils has been described. This model has included a specific description of the role of cohesion, a capability to deal with sediment with a range of sizes and thus settling velocities, and an explicit representation of the layer formed by deposition. By delineating between the processes of entrainment, which acts upon the original cohesive soil, and reentrainment, which acts upon the deposited layer, a physical description of the erosion of cohesive soils has been provided. When cohesion plays no role in limiting the transport of sediment, the derived expression for sediment concentration has been shown to have similarities to the sediment transport equation of *Yang* [1973]. Finally, the model was shown to have good agreement with the slope length term used in the universal soil loss equation.

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