D 常用數學公式

D.1 三角函數

$\text{Sin}\theta = 1/\text{Csc}\theta$

$\text{Cos}\theta = 1/\text{Sec}\theta$

$\text{Tan}\theta = \text{Sin}\theta / \text{Cos}\theta$

$\text{Sin}^2\theta + \text{Cos}^2\theta = 1$

$\text{Sec}^2\theta - \text{Tan}^2\theta = 1$

$\text{Csc}^2\theta - \text{Cot}^2\theta = 1$

$\text{Sin}^2\theta = \frac{1}{2} - \frac{1}{2}\text{Cos}2\theta$

$\text{Sin}^2\theta = 1 - \text{Cos}^2\theta$

$1 - \text{Cos}^2\theta = \frac{1}{2} - \frac{1}{2}\text{Cos}2\theta$

$\text{Cos}^2\theta = \frac{1}{2} + \frac{1}{2}\text{Cos}2\theta$

$\text{Cos}2\theta = \text{Cos}^2\theta - \text{Sin}^2\theta$

$\text{Sin}2\theta = 2\text{Cos}\theta\,\text{Sin}\theta$

$\text{SinX SinY} = \frac{1}{2}[\text{Cos}(X-Y) - \text{Cos}(X+Y)]$

$\text{CosX CosY} = \frac{1}{2}[\text{Cos}(X-Y) + \text{Cos}(X+Y)]$

$\text{SinX CosY} = \frac{1}{2}[\text{Sin}(X-Y) + \text{Sin}(X+Y)]$

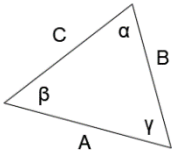
Cos(X+Y)= CosX CosY - SinX SinY

Cos(X-Y)= CosX CosY + SinX SinY

Sin(X+Y)= SinX CosY + SinY CosX

Sin(X-Y)= SinX CosY - SinY CosX

$\frac{A}{Sin\alpha} = \frac{B}{Sin\beta} = \frac{C}{Sin\gamma}$



P=(A+B+C)/2

面積=$[P(P-A)(P-B)(P-C)]^{1/2}$

$A^2=B^2+C^2-2BC\ Cos\alpha$

$\frac{1-Sin\phi}{1+\sin\phi} = \tan^2(45-\frac{\phi}{2})$　主動側土壓

$\frac{1-Sin\phi}{1+\sin\phi} = \tan^2(45+\frac{\phi}{2})$　被動側土壓

$1-\sin\phi$　靜止動側土壓

D.2 微分

$\frac{dx^n}{\mathrm{dx}} = nx^{n-1}$

$\frac{d}{dx}Sinx = Cosx$

$\frac{d}{dx}Cosx = -Sinx$

$\frac{d}{dx}Tanx = \mathrm{Sec}^2x$

$\frac{d}{dx}\mathrm{Sin}^{-1}x = 1/\sqrt{(1-\mathrm{x}^2)}$

$\frac{d}{dx}\mathrm{Cos}^{-1}x = -1/\sqrt{(1-\mathrm{x}^2)}$

$\frac{d}{dx}\mathrm{Tan}^{-1}x = 1/(1-\mathrm{x}^2)$

$\mathrm{f'(x)}=\frac{df(x)}{dx}$

(f+g)'(x)=f'(x)+g'(x)

(fg)'(x)=f(x)g'(x)+f'(x)g(x)

$(\frac{1}{g})'(x) = \frac{g'(x)}{[g(x)]^2}$

$(\frac{f}{g})'(x) = \frac{\mathrm{f'(x)g(x)-f(x)g'(x)}}{[g(x)]^2}$

D.3 積分

$$\int adx = ax + c$$

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1} + c$$

$\int \frac{1}{x} dx = \ln x + c$

$$\int e^x dx = e^x + c$$

$$\int Sinxdx = -Cosx + c$$

$$\int Cosxdx = Sinx + c$$

$$\int Sin\, mx \, dx = -\frac{1}{m} Cos\, mx + c$$

$$\int Cos\, mx \, dx = \frac{1}{m} Sin\, mx + c$$

$$\int Sin^m x \, Cosx \, dx = \frac{1}{m+1} Sin^{m+1} x + c$$

$$\int Cos^m x \, Sinx \, dx = \frac{-1}{m+1} Cos^{m+1} x + c$$

$\int (Sin^m x)(Cos^m x)dx$

m & n 是偶數

利用 $Sin^2\theta = \frac{1}{2} - \frac{1}{2}Cos2\theta$ 展開

$Cos^2\theta = \frac{1}{2} + \frac{1}{2}Cos2\theta$

$\int Sin(mx)Cos(nx)dx$

$\int Sin(mx)Sin(nx)dx$

$\int Cos(mx)Cos(nx)dx$

利用 $SinX\ SinY = \frac{1}{2}[Cos(X-Y)-Cos(X+Y)]$ 展開

$CosX\ CosY = \frac{1}{2}[Cos(X-Y)+Cos(X+Y)]$

$SinX\ CosY = \frac{1}{2}[Sin(X-Y)+Sin(X+Y)]$

$\int (Sin^m x)(Cos^m x)dx$

m or n 是奇數

利用 $Sin^2\theta + Cos^2\theta = 1$

$Sin^2\theta = 1 - Cos^2\theta$ 展開

$Cos^2\theta = 1 - Sin^2\theta$

$\int Sin^3 x dx = \int Sinx\ dx - \int Cos^2 x\ Sinx\ dx$

$= -cosx + \frac{1}{3}Cos^3 x$

$Sin^3 x = Sin^2 x\ Sinx$

$= (1 - Cos^2 x)Sinx$

$= Sinx - Cos^2 x Sinx$

$\int Cos^2 x\ Sinx\ dx = \int -u^2\ du = -\frac{1}{3}u^3 + c$

$u = Cosx$

$du = -sinxdx$

$u^2 = Cos^2 x$

圓弧結構常用之積分

$$\int Sin2\phi = -\frac{1}{2}Cos2\phi + c$$

$$\int Cos2\phi = \frac{1}{2}Sin2\phi + c$$

$$\int Sin^2\phi = \frac{1}{2}(\phi + Sin2\phi) + c$$

$$\int (1 - Cos\phi)^2 = \frac{1}{2}(3\phi - 4Sin\phi - Sin2\phi) + c$$

$$\int (1 - Cos\phi) = (\phi - Sin\phi)$$

上之公式由下列導出

$$\mathrm{Sin}^2\theta = \frac{1}{2} - \frac{1}{2}\mathrm{Cos}2\theta$$

$$\mathrm{Cos}^2\theta = \frac{1}{2} + \frac{1}{2}\mathrm{Cos}2\theta$$

$$\int Sin\, mx\, dx = -\frac{1}{m}Cos\, mx + c$$

$$\int Cos\, mx\, dx = \frac{1}{m}Sin\, mx + c$$

積分 Chain Rule

$$\int Sin(mx)dx = \frac{-1}{m}Cos(mx) + c$$

→u=mx, du=mdx dx=du/m

$$\rightarrow \int \frac{1}{m}Sin\,u\,du = \frac{-1}{m}Cos\,u\,du = \frac{-1}{m}Cos\,mx + c$$

$$\int Cos(mx)dx = \frac{1}{m}Sin\,(mx) + c$$

$$\rightarrow \int \frac{1}{m}Cos\,u\,du = \frac{1}{m}Sin\,u\,du = \frac{1}{m}Sin\,mx + c$$

$$\int Sin^m x\,Cos\,x dx = \frac{1}{m+1}Sin^{m+1}x+c$$

→u=Sinx, du=Cosxdx dx=du/Cosx

$$\rightarrow \int \frac{u^m Cos\,x}{Cosx}du = \int u^m\,du = \frac{1}{m+1}u^{m+1} + c = \frac{1}{m+1}Sin^{m+1}x+c$$

$$\int Cos^m x\,Sin\,x dx = \frac{-1}{m+1}Cos^{m+1}x+c$$

→u=Cosx, du=-Sinxdx dx=-du/Sinx

$$\rightarrow \int -\frac{u^m Sinx}{Sinx}du = \int -u^m\,du = \frac{-1}{m+1}u^{m+1} + c =$$

$$\frac{-1}{m+1}Cos^{m+1}x+c$$